Coupling coefficients and propagation constants in guided wave distributed-feedback lasers
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Coupling coefficients and propagation constants in guided wave distributed-feedback lasers

William Streifer, Robert D. Burnham, and Donald R. Scifres

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Coupling coefficients and propagation constants in guided-wave distributed-feedback lasers are calculated. It is shown that higher-order transverse modes often have substantially larger coupling coefficients than do low-order transverse modes. Thus, higher-order modes require lower net gains at threshold. Grating spacings which resonate these higher-order modes are calculated and shown to differ significantly from the values appropriate to low-order modes. Higher-order modes also resonate at greater frequency separations, thus facilitating single-mode operation.

In optimally designing distributed-feedback (DFB) lasers (both junction and other types) it is important to determine coupling coefficients and propagation constants. Coupling coefficients \( \kappa \) determine the minimum net gain and/or length of the structure required to initiate laser operation (see Kogelnik and Shank\(^3\) and Chinn\(^4\)); overly large values of \( \kappa L \) (\( L \) being the length) may also be suboptimal.\(^5,6\) Propagation constants \( \beta \) determine the transverse-mode separation and more importantly the required period of the DFB structure. In this communication we present results for \( \kappa \) and \( \beta \) as a function of the parameters in three-layer guiding structures, which encompass single-heterojunction (SH) diodes, double-heterojunction (DH) diodes, and other thin film lasers. It is shown that in many cases higher-order transverse modes have substantially higher coupling coefficients; to exploit this fact grating periods must be chosen which differ significantly from those usually utilized (see also Refs. 5 and 6).

The particular geometry analyzed is illustrated in Fig. 1(a) without the DFB structure, which in turn is illustrated in Fig. 1(b). Our calculations of \( \beta \) and \( \kappa \) are similar to those of Yariv\(^7\) with some obvious notational changes. For TE modes

\[
\beta = \left( n^2_2 k^2 - \beta^2 \right)^{1/2},
\]

\[
q = (\beta^2 - n^2_2 k^2)^{1/2},
\]

\[
p = (\beta^2 - n^2_2 k^2)^{1/2},
\]

\[
\tan(hq) = \frac{h(p + q)}{(h^2 - p^2)},
\]

where \( k = \omega/c \) and the coupling coefficient for a TE mode is

\[
\kappa = k^2 \left( \frac{n_2^2 - n_3^2}{m \pi \beta} \right) \frac{h^2}{(h^2 + q^2)(l + q^2 + p^2)} \times \int_0^\infty \left( \cos(hx) + \frac{q}{h} \sin(hx) \right) \sin \left( \frac{m \pi w(x)}{\Lambda} \right) dx,
\]

where all symbols are defined above or in Fig. 1 and \( m \) is the grating order, i.e., \( \beta = n \pi \Lambda / \Lambda \). For a rectangular grating with \( w(x) = w_0 \) for \( 0 < x < \Lambda \), Eq. (2) becomes

\[
\kappa = \frac{k^2 (n_2^2 - n_3^2)}{m \pi \beta} \frac{h^2}{(h^2 + q^2)(l + q^2 + p^2)} \sin \left( \frac{m \pi w(x)}{\Lambda} \right) \times \left[ g + \frac{\sin(2gh)}{2h} + \frac{q}{h^2} \left( 1 - \cos(2gh) \right) + \frac{q^2}{h^2} \left( g - \frac{\sin(2gh)}{2h} \right) \right].
\]

First consider \( \beta \), obtained by numerically solving Eqs. (1a)–(1d). Note that the three-layer structure itself, with no grating as shown in Fig. 1(a), deter-

<table>
<thead>
<tr>
<th>( t (\mu m) )</th>
<th>Mode No.</th>
<th>( \beta (\mu m^{-1}) )</th>
<th>( \lambda_e (\mu m) )</th>
<th>( \Lambda (\AA) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1</td>
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<td>0.2374</td>
<td>3561</td>
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<tr>
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<td>1</td>
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<td>0.2368</td>
<td>3552</td>
</tr>
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<td>0.2365</td>
<td>3548</td>
</tr>
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<td>3546</td>
</tr>
<tr>
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<td>26.597</td>
<td>0.2371</td>
<td>3557</td>
</tr>
<tr>
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<td>2</td>
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<td>0.2363</td>
<td>3544</td>
</tr>
<tr>
<td>2.5</td>
<td>1</td>
<td>26.590</td>
<td>0.2368</td>
<td>3552</td>
</tr>
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<td>0.2362</td>
<td>3544</td>
</tr>
<tr>
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<td>2</td>
<td>26.481</td>
<td>0.2373</td>
<td>3550</td>
</tr>
</tbody>
</table>

TABLE I. TE mode propagation constants in a SH diode for various layer thickness and \( \lambda_e = 6500 \AA \).

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we utilize the results of Ref. 3 in which it is shown that the lowest-threshold longitudinal-mode resonance shifts negligibly from the grating, i.e.,

$$\left(\beta - \lambda/3\pi\right) \ll \beta.$$  \hfill (6)

Since the actual physical grating period \(\Lambda\) is fixed in a particular laser it is important to compute \(\lambda_0\) given \(\Lambda\). Let \(\Lambda = 3546\ \text{Å}\), chosen to resonate the first mode for \(t = 2\ \mu\text{m}\) with \(\lambda_0 = 8500\ \text{Å}\) output. Then if the second mode were to oscillate with the same \(\lambda_0\) (and therefore \(\beta\)) as determined by the DFB grating, its output wavelength would be approximately (cf. Table I)

$$\lambda_0^{(2)} = (8500/2371)2364\ \text{Å} = 8475\ \text{Å},$$

which is shifted by 25 Å from the first mode. In Fig. 2, \(\kappa\) vs \(t\) is plotted for the same SH structure with \(\lambda_0 = 8500\ \text{Å}\) and \(\omega_0/\lambda = 0.25\), for third-order operation. There

![FIG. 2. Coupling coefficient vs thickness of gain region for a single-heterojunction geometry.](image)

![FIG. 3. Coupling coefficient vs grating height for a double-heterojunction geometry.](image)

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### TABLE II. TE mode propagation constants in a DH diode for \(t = 2\ \mu\text{m}\) and \(\lambda_0 = 8500\ \text{Å}\).

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>(\beta) ((\mu\text{m}^{-1}))</th>
<th>(\lambda_0) ((\mu\text{m}))</th>
<th>(\Lambda) (Å)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>26.574</td>
<td>0.2364</td>
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<tr>
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<td>0.2374</td>
<td>3562</td>
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<tr>
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<td>0.2391</td>
<td>3587</td>
</tr>
<tr>
<td>4</td>
<td>26.017</td>
<td>0.2415</td>
<td>3623</td>
</tr>
<tr>
<td>5</td>
<td>25.691</td>
<td>0.2446</td>
<td>3669</td>
</tr>
<tr>
<td>6</td>
<td>25.314</td>
<td>0.2482</td>
<td>3723</td>
</tr>
</tbody>
</table>

---

### TABLE III. Free-space wavelengths for various grating periods in a DH structure.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>(\lambda_0) (Å)</th>
<th>(\lambda_0) (Å)</th>
<th>(\lambda_0) (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3547</td>
<td>3623</td>
<td>3669</td>
</tr>
<tr>
<td>2</td>
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<td>3546</td>
<td>3621</td>
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<tr>
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<td>3546</td>
<td>3621</td>
<td>3669</td>
</tr>
<tr>
<td>6</td>
<td>3546</td>
<td>3621</td>
<td>3669</td>
</tr>
</tbody>
</table>

---
exist large ranges of $t$ for which $\kappa_2 >> \kappa_1$. Specifically, for $t = 2 \mu m$, $\kappa_1 = 7.61 \times 10^{-4} \mu m^{-1}$ and $\kappa_2 = 2.40 \times 10^{-3} \mu m^{-1}$ so that the second mode will have a much lower threshold than the first.

Consider next a DH diode with $n_1 = 3.4$, $n_2 = 3.6$, $n_3 = 3.4$, and $t = 2 \mu m$. We calculate $\beta$ and then $\kappa$; the latter as a function of rectangular grating height $g$ for $n/\lambda = 0.25$ and third-order operation. Values of $\beta$, $\lambda_0$, and $\Lambda$ are listed in Table II for $\lambda_0 = 8500 \AA$. Clearly, for particular values of $\lambda$ each of the transverse modes resonates at a different free-space wavelength $\lambda_0$. Values of $\lambda_0$ for three different grating spacings chosen to resonate the first, fourth, and fifth modes, respectively, at $\lambda_0 = 8500 \AA$ are listed in Table III. We note that for $\Lambda = 3623$ and 3669 $\AA$ the modes adjacent to those resonant at $\lambda_0 = 8500 \AA$ are shifted so far from line center that they experience substantially reduced net gain.

Figure 3 is a plot of $\kappa$ vs grating height for propagating modes in the DH geometry. Generally, $\kappa$ increases with mode number, which reflects the fact that the higher modes have larger relative amplitudes in the vicinity of the grating. Also, $\kappa$ increases with grating height $g$; however, when $g$ approximates the zero of a particular mode $\partial \kappa / \partial g = 0$. This occurs for the sixth mode at $g \approx 4500 \AA$; $\kappa$ does increase for that mode with further increases in $g$ and, in fact, $\kappa_6$ exceeds $\kappa_5$ at $g \approx 4500 \AA$. Clearly, $\kappa_4$, $\kappa_5$, and $\kappa_6$ are substantially larger than $\kappa_1$ or $\kappa_2$ for small values of $g$. For $g = 1500 \AA$, for example, $\kappa_4$ and $\kappa_5$ are over one order of magnitude greater than $\kappa_1$. Thus, a DH diode laser should have substantially lower threshold with $\Lambda = 3623$ or $3669 \AA$ than with $\Lambda = 3548 \AA$. A concomitant advantage of laser operation with these grating periods is the large transverse-mode separation.

Identical calculations have been carried out for TM modes with very similar results. Generally, $\kappa$ for TM modes is slightly smaller than that for corresponding TE modes, but the differences are not significant. It should also be noted that the above analysis is based on perturbations rather than an exact solution of the boundary value problem (with grating present). We believe the results to be valid for the grating heights and structural dimensions considered herein.

In conclusion, we have shown that higher-order transverse modes in guided-wave structures often have much larger coupling coefficients than do lower modes. Grating spacings required to resonate the higher-order transverse modes have been calculated and these modes were shown to have large separations in frequency, thus facilitating single-mode operation. The results indicate that it is often desirable to fabricate the DFB grating of a guided-wave laser at a period which differs substantially from that required to resonate the lowest-order mode.